

Name:

Spring 2020 Stat 311 Exam 1

Instructions: WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. SHOW YOUR WORK!

1. 4 families bring their kids to the central park merry-go-round. Families A and B have 3 kids each and families C and D have 2 kids each. Siblings must sit together. How many sitting arrangements are possible if, to complicate things, the A siblings refuse to sit next to the B siblings? [10 pts]

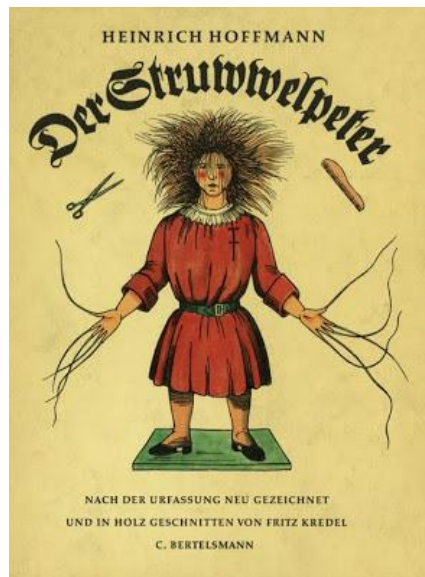
2. In a frozen yogurt parlor you can order from among 6 yogurt types and put on any number of 10 available toppings (no toppings is plainly an option). If the yoghurt comes in 3 types of cups (small, medium, large), how many different orders are possible? [10 pts]

3. There are 16 balls in a jar, g of which are green and r -red ($g + r = 16$). Two balls are randomly sampled in a sequence (i.e. the balls are removed one by one). If the probability of drawing two balls of the same color equals the probability of drawing two different colors, what are the possibilities for g and r ? [10 pts]
4. 5 regular decks of playing cards are mixed, making a super-deck of $5 \cdot 52$ cards. Clearly each card type repeats 5 times within the super-deck. 5 cards are then given to a player. How many unordered 5 card hands are possible? [10 pts]

5. 6 cards are randomly selected from among an ordinary deck of 52 cards. What is the probability of being dealt 3 pairs? (This occurs when the cards have denominations a, a, b, b, c, c and a, b, c are all distinct). [10 pts]

6. A *norepeatword* is a an ordered sequence of letters none of which repeats (e.g. "course" and "source" are norepeatwords but "necessary" isn't). A nonrepeatword is chosen randomly, with all norepeatwords equally likely. Show that the probability that it uses all 26 letters is very close to $1/e$ [10 pts]

7. A pair of dice is rolled until a sum of either 4 or 8 appears. Find the probability that a 4 occurs first [10 pts]
8. Struwwelpeter's wardrobe has 4 pairs of shoes and 3 pairs of socks (green socks, red socks, and blue socks). Being characteristically disorganized, he picks two shoes and two socks at random. What is the probability that the shoes make a complete pair and the socks match in color?



[10 pts]

9. 37 probability students are anxious about the upcoming exam. They all attend the review on Monday and all come to take the Wednesday test. If the instructor randomly assigns their seats, what is a good approximation to the probability that no student occupies the same seat as on Monday? Explain. (Assume there are exactly 37 seats in the classroom) [10 pts]
10. 5 chess players will each play one match against the other. How many outcomes are possible if each outcome is to record the result of every match? Each match can result in a win for one player or else it is a draw. [10 pts]

Extra Credit

11. Find a formula for calculating the probability that the resultant sum upon rolling 4 fair dice is n for $4 \leq n \leq 48$ [10 pts]

12. Give a combinatorial argument for the identity $\sum_{k=0}^j \binom{n}{k} \binom{m}{j-k} = \binom{m+n}{j}$ [10 pts]

13. In Dostoevsky's "The Gambler", who is called utchitel and why?

[10 pts]